

### MAT 1320 - Final Exam

You're out for a Sunday drive with your significant other. (Does anyone actually go for Sunday drives anymore?)

1. Your car's location with respect to time can be represented by the function  
 $p(t) = t^3 + t^2 - t + 3$ .
  - a. What is the initial location of the car?
  - b. What is the velocity at the car at  $t=1$ ?
  - c. What is the acceleration of the car at  $t=4$ ?
2. During your drive, you get stuck in construction. You speak to one of the workers, and they say there's a bridge being constructed up ahead, but they're having trouble figuring out how to connect the bridge to the road on the other side. To do this, they need to figure out the value of  $a$  that satisfies both equations:

$$f(x) = \begin{cases} \text{Bridge} & x^2 - 1 & -1 \leq x < 2 \\ \text{Road} & a + 3x & x \geq 2 \end{cases}$$

Care to help them out?

3. While you're helping the construction workers, your SO decides to crack open the newspaper and check out the puzzle section. It looks like a little different this week.
  - a. Evaluate:
    - i.  $\lim_{t \rightarrow 2} \frac{t^2 - 4}{t - 2}$
    - ii.  $\lim_{x \rightarrow 0} \frac{x^2 + 3x}{3x^2 - 3x}$
  - b. Find  $dy/dx$  for the following functions:
    - i.  $y = x^3 + 3x^2 + \frac{x}{\sin x}$
    - ii.  $x \ln y + \frac{3}{x} = y^3 - x$
    - iii.  $y = x^{\sin x}$
  - c. Evaluate the following integrals:
    - i.  $\int \left( 4x^3 - \frac{12}{x-1} + \sqrt{x} \right) dx$
    - ii.  $\int \frac{12}{4 + 4x^2} dx$
    - iii.  $\int_0^1 x e^{2x} dx$
    - iv.  $\int x^2 \sin x dx$

$$\text{v. } \int \frac{42x}{(2+9x^2)^3} dx$$

$$\text{vi. } \int_0^1 x^9 \sqrt{1-x^{10}} dx$$

4. The construction workers have one last thing to ask you before you leave. They have to install build a temporary control box near the bridge to do... stuff... It must have the side of a square-based prism, with an open-top. It also must have a volume of  $1000 \text{ cm}^3$ . Since wood costs a lot these days, find the dimensions of this control box so that the surface area is minimized.
5. Now that you're done with the construction workers, you head back out on your drive. There's only a toll road left before you get back home. You can either choose to pay \$100 (WHAT?!) or solve the following problem:

*The following left-hand Riemann sum is defined for  $f(x)$  on the interval  $[0,5]$ .*

*a) Determine  $f(x)$ .*

*b) Evaluate the limit of the Riemann sum.*

$$\lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} \frac{5}{n} \left[ \left( \frac{5i}{n} \right)^3 + \frac{5i}{n} \right]$$

After all those odd experiences, you're pretty sure you won't go on a Sunday drive again. Best to just stay home and watch Netflix. Much more productive.